



Rewarding Learning

General Certificate of Secondary Education
2025

Centre Number

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Candidate Number

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Further Mathematics

Unit 1 (With calculator)

Pure Mathematics



[GFM11]

GFM11

MONDAY 12 MAY, AFTERNOON

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.

You must answer the questions in the spaces provided.

Do not write outside the boxed area on each page or on blank pages.

Complete in black ink only. **Do not write with a gel pen.**

Questions which require drawing can be completed using an HB pencil.

All working **must** be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.

Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.

Answer **all fourteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

You may use a calculator.

The Formula Sheet is on page 2.

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Formula Sheet

PURE MATHEMATICS

Quadratic equations: If $ax^2 + bx + c = 0$ ($a \neq 0$)

$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Differentiation: If $y = ax^n$ then $\frac{dy}{dx} = nax^{n-1}$

Integration: $\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$ ($n \neq -1$)

Logarithms: If $a^x = n$ then $x = \log_a n$

$$\log(ab) = \log a + \log b$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$$\log a^n = n \log a$$

Matrices:

$$\text{If } \mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{then } \det \mathbf{A} = ad - bc$$

$$\text{and } \mathbf{A}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad (ad - bc \neq 0)$$



1 Matrices **X**, **Y** and **Z** are defined by

$$\mathbf{X} = \begin{bmatrix} -2 & 7 \\ 4 & -3 \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} \quad \text{and} \quad \mathbf{Z} = \begin{bmatrix} -2 & 3 & -2 \\ 5 & -1 & 4 \end{bmatrix}$$

Calculate

(i) $\mathbf{X} + 2\mathbf{Y}$

Answer _____ [2]

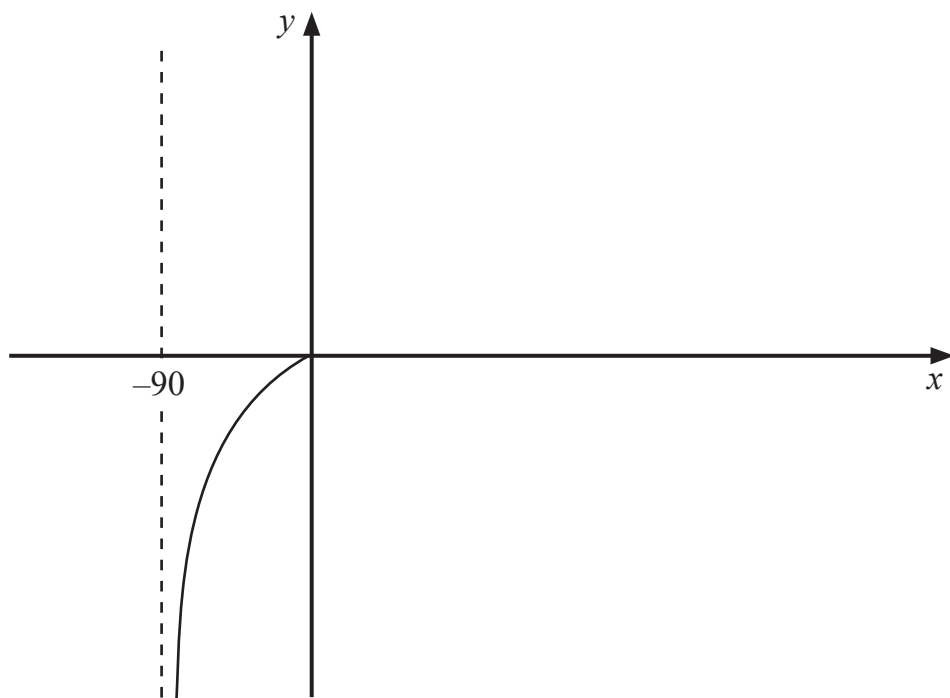
(ii) \mathbf{XZ}

Answer _____ [2]

[Turn over



- 2 (a) Complete the graph of $y = \tan x$ for $-90^\circ \leq x \leq 270^\circ$ on the axes below.



[1]



(b) (i) Solve the equation

$$\tan x = 6$$

$$\text{for } -90^\circ \leq x \leq 270^\circ$$

Answer _____ [2]

(ii) Hence solve the equation

$$\tan(3\theta + 45^\circ) = 6$$

$$\text{for } -30^\circ \leq \theta \leq 90^\circ$$

Answer _____ [3]

[Turn over



3 A function $f(x)$ is defined by

$$f(x) = x^2 - 3x + 5$$

(i) Use the method of **completing the square** to rewrite $f(x)$ in the form

$$(x + a)^2 + b$$

where a and b are constants.

Answer _____ [2]

(ii) Hence write down the coordinates of the minimum turning point of the curve

$$y = x^2 - 3x + 5$$

Answer _____ [2]



4 Solve the inequality

$$2x^2 - 25 \leq 7x + 5$$

You **must** show clearly each stage of your solution.

Answer _____ [5]

[Turn over

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5 (i) Find \mathbf{B}^{-1} where $\mathbf{B} = \begin{bmatrix} 5 & 2 \\ 3 & -4 \end{bmatrix}$

Answer _____ [2]



(ii) Hence using a matrix method, solve the following simultaneous equations for x and y .

$$5x + 2y = 4$$

$$3x - 4y = 18$$

Answer $x =$ _____ , $y =$ _____ [4]

[Turn over



6 The gradient function of a curve is given by

$$\frac{dy}{dx} = 6x + \frac{1}{x^3}$$

The curve passes through the point (1, 5).

Find the equation of the curve.

Answer _____ [4]



7 Simplify **fully** the following algebraic expression.

$$\left(\frac{x}{x-3} - \frac{x+4}{x+3} \right) \div \frac{x+6}{x^2-9}$$

Answer _____ [7]

[Turn over

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8 $y = \frac{4}{7x^3}$

(i) Find $\frac{dy}{dx}$

Answer _____ [1]

(ii) Hence, using your answer to part (i), solve the equation

$$y = -2 \frac{dy}{dx}$$

Answer $x =$ _____ [2]





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(Questions continue overleaf)

[Turn over

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9 (a) Express as a single logarithm

$$\log a - 4\log\sqrt{b} + \log c$$

Answer _____ [3]



(b) Solve the equation

$$5^{3x+1} = 7^{2-x}$$

Answer _____ [5]

[Turn over

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32GFM1115

- 10 Tommy recorded the time, T hours, he spent revising the night before each of his Further Mathematics tests and the mark, M %, he achieved.

The results are shown in the table below.

Time T hours	Mark M %		
2.75	88		
2.36	83		
2	78		
1.4	68		
1.2	64		

Tommy believes that a relationship of the form

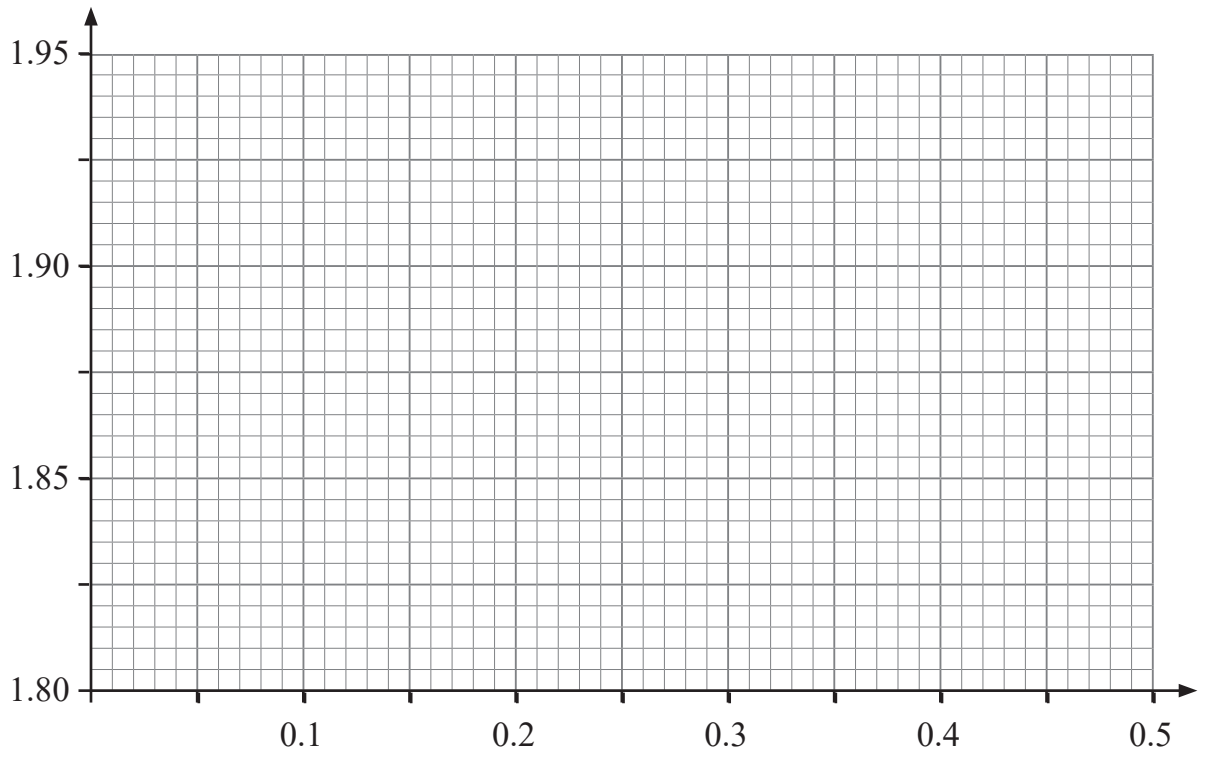
$$M = kT^v$$

exists, where k and v are constants.

- (i) Verify that a relationship of the form $M = kT^v$ exists by drawing a suitable straight line graph on the grid opposite. Label the axes clearly.

Show clearly the values used, correct to 3 decimal places, in the table above.





[6]

[Turn over

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(ii) Hence find the values of k and v , correct to 2 decimal places.

Answer $k =$ _____ , $v =$ _____ [4]



(iii) Show clearly, using your values for k and v , that the relationship $M = kT^v$ does **not** hold if Tommy revises for 4 hours.

[2]



11 A curve is defined by the equation $y = 4x^2 + 3x - 1$

(i) Find the **coordinates** of the points where the curve meets the x -axis.

Answer _____ [2]

(ii) Write down the coordinates of the point where the curve meets the y -axis.

Answer _____ [1]



(iii) Using calculus, find the coordinates of the turning point of the curve.

Answer _____ [3]

[Turn over

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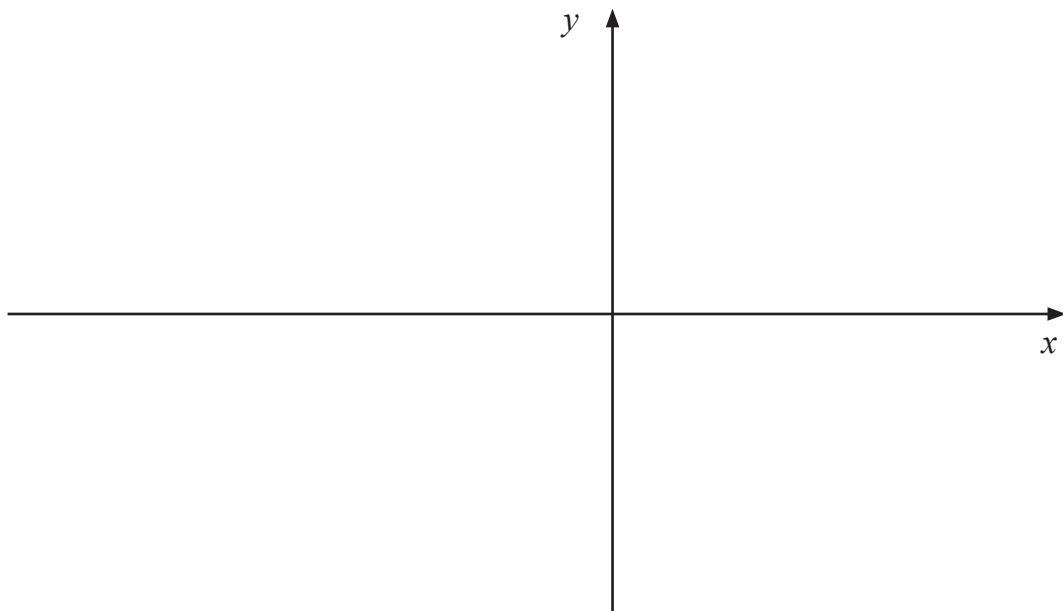
(iv) Show clearly why this turning point is a minimum.

[1]

(v) Hence, using your answers from parts (i) to (iv), sketch the curve of

$$y = 4x^2 + 3x - 1$$

on the axes below.



[2]



(vi) Find the area enclosed by the curve, the x -axis and the lines $x = -\frac{1}{2}$ and $x = 0$

Answer _____ [4]

[Turn over



12 Andy, Bella and Conrad have been on a trip together to Germany, Poland and Sweden.

They need to convert their remaining foreign currency into pounds (£) after coming home.

The table below shows how much of each foreign currency they returned with.

	Euro (Germany)	Zloty (Poland)	Krona (Sweden)
Andy	1200	700	600
Bella	300	2700	800
Conrad	800	2500	500

For example, Andy returned with 1200 euro, 700 zloty and 600 krona on 16th April.

Let x , y and z represent the rate, in pounds, for each euro, zloty and krona converted respectively on 16th April.

Andy converted his foreign currency and received £1,200 on 16th April.

This can be represented by the equation

$$12x + 7y + 6z = 12$$



- (i) Bella converted her foreign currency and received £800 on 16th April.

Show that

$$3x + 27y + 8z = 8$$

[1]

Conrad converted his foreign currency a month later when the rate, in pounds, for each zloty had decreased by 40% and the rate for each krona had increased by 20%. He received £1,000

- (ii) Show that

$$8x + 15y + 6z = 10$$

[2]

[Turn over



(iii) Solve the equations

$$12x + 7y + 6z = 12$$

$$3x + 27y + 8z = 8$$

$$8x + 15y + 6z = 10$$

to find the values of x , y and z .

Show clearly each stage of your solution.



Answer $x =$ _____, $y =$ _____, $z =$ _____ [8]

- (iv) On the same day that Conrad converted his foreign currency, Dillon converted £150 into Polish zloty.

How much did Dillon receive?

Answer _____ zloty [2]

[Turn over

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13 A curve is defined by the equation

$$y = 3x^2 - 5x - 1$$

The gradient of the tangent to the curve at the point P is equal to 5

(i) Find the equation of the tangent to the curve at the point P.

Answer _____ [5]



The point Q also lies on the curve.

The tangent to the curve at P is perpendicular to the tangent at Q.

(ii) Find the coordinates of Q.

Answer _____ [3]

[Turn over

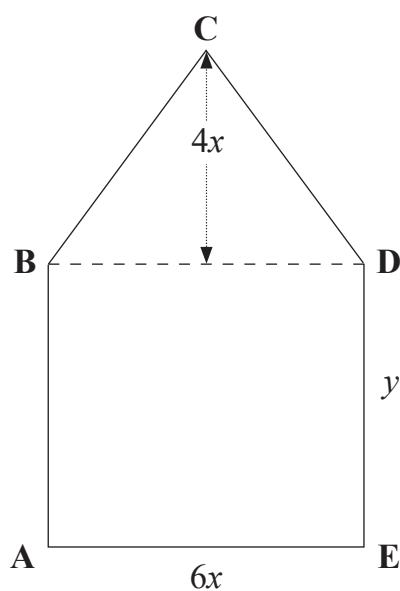
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- 14 A poster designer is making a sign for a client in the shape of an isosceles triangle on top of a rectangle.

The design with dimensions, in metres, is shown in the diagram below.



The area of the rectangle ABDE is 12 m^2

- (i) Show that $y = \frac{2}{x}$

[1]



(ii) The length of BC is $5x$

Show that the expression for the perimeter of the sign can be written as

$$P = 16x + \frac{4}{x}$$

[1]

(iii) Hence, using calculus, find the value of x that will give the minimum possible perimeter of the sign, showing that it is a minimum.

Answer $x =$ _____ [5]



THIS IS THE END OF THE QUESTION PAPER

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For Examiner's use only	
Question Number	Marks
1	
2	
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14	

Total Marks	
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Examiner Number

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